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SITE INDEX CURVES FOR ENGELMANN SPRUCE IN THE NORTHERN AND CENTRAL ROCKY MOUNTAINS

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ABSTRACT

This paper presents in graphic and tabular form a set of polymorphic site index curves for Engelmann spruce. The curves are based on a new generalization of the Osborne-Schumacher method of site curve construction. A table of standard errors of the estimate obtained by given numbers of sample trees per plot will help the user establish desired sample size. Approximation equations are also given so these polymorphic site curves can be used in computer programs. The basic data came from the northern and central Rocky Mountain area, but a statistical test with limited data from Arizona and New Mexico indicates the site index curves may also be valid for the southwestern States.

INTRODUCTION

This paper presents a new system of polymorphic site index curves for Engelmann spruce (Picea engelmanni Parry) in the Mountain States. Polymorphism in a set of curves means that the shape of the curves varies from one level of site index to another; that is, the curves are not proportional, in contrast to anamorphic or proportional curves. Differences between polymorphic and anamorphic site index curves, and the superiority of the former, are discussed in the standard forest mensuration texts (Husch 1963; Spurr 1952). Two equations with which electronic computers can be used to replace manual methods of site index assignment are also given.

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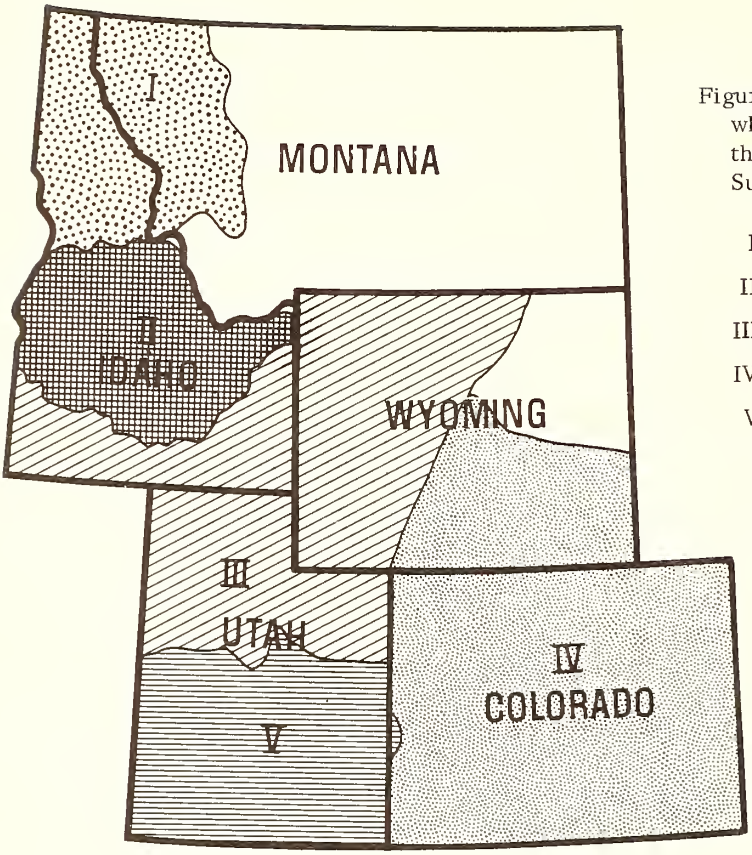

Figure 1.--The area from which the data used in this study were drawn. Subregions are:

SOURCE OF DATA

The data upon which the construction of these curves was based are measurements of total age and total height for 1,928 dominant and codominant spruce trees. These data were collected by Forest Survey in the course of forest inventories over the past 10 years. The geographical area sampled is shown in figure 1. This area was divided into five subregions, differing from one to another with respect to general climatic conditions that might influence tree growth (U.S. Department of Agriculture 1941). From one to three trees were measured on each Forest Survey sampling location[2] which fell within the Engelmann spruce type. Total tree age was obtained by counting annual rings on an increment core taken at breast height and adding to this the estimated time required for the tree to reach breast height. Data of this kind are, of course, inferior to repeated height measurements on permanent plots or to stem analysis data. Nevertheless, because a need has long existed for a means of classifying Engelmann spruce stands according to site quality, the data available were used in such a way as to extract as much information as possible.

SITE INDEX CURVES AND TABLE

The site index curves resulting from this study are shown in figure 2. The index base age is 50 years.[3] Table 1 shows the expected height of trees in the dominant stand according to total age and site index.

[2] As used by Forest Survey, "location" refers to a cluster of either two or three fixed area or 10 variable radius plots. Fixed area plots were either one-fourth or one-fifth acre in size.

[3] The base age is established arbitrarily and in no way affects the shape of the resulting curves.

HOW TO USE THIS INFORMATION

To estimate the average site index for a stand of Engelmann spruce, measure total age and total height of trees that are in the dominant portion of the stand. Site trees should also give evidence of having been in the dominant stand throughout their lives. Table 2 shows the sample size expected to be necessary on plots up to 1 acre in size in order to attain a given standard error of the estimated mean plot site index. These estimates of required sample size are based on the variance of site index between individual trees within forest survey plot clusters on homogeneous sites. As plot size decreases below one-fourth acre, a somewhat smaller sample may achieve the same precision. Using figure 2 or table 1, estimate a site index value for each sample tree. The average site index for all sample trees will be the best estimate of average site index for the area sampled.

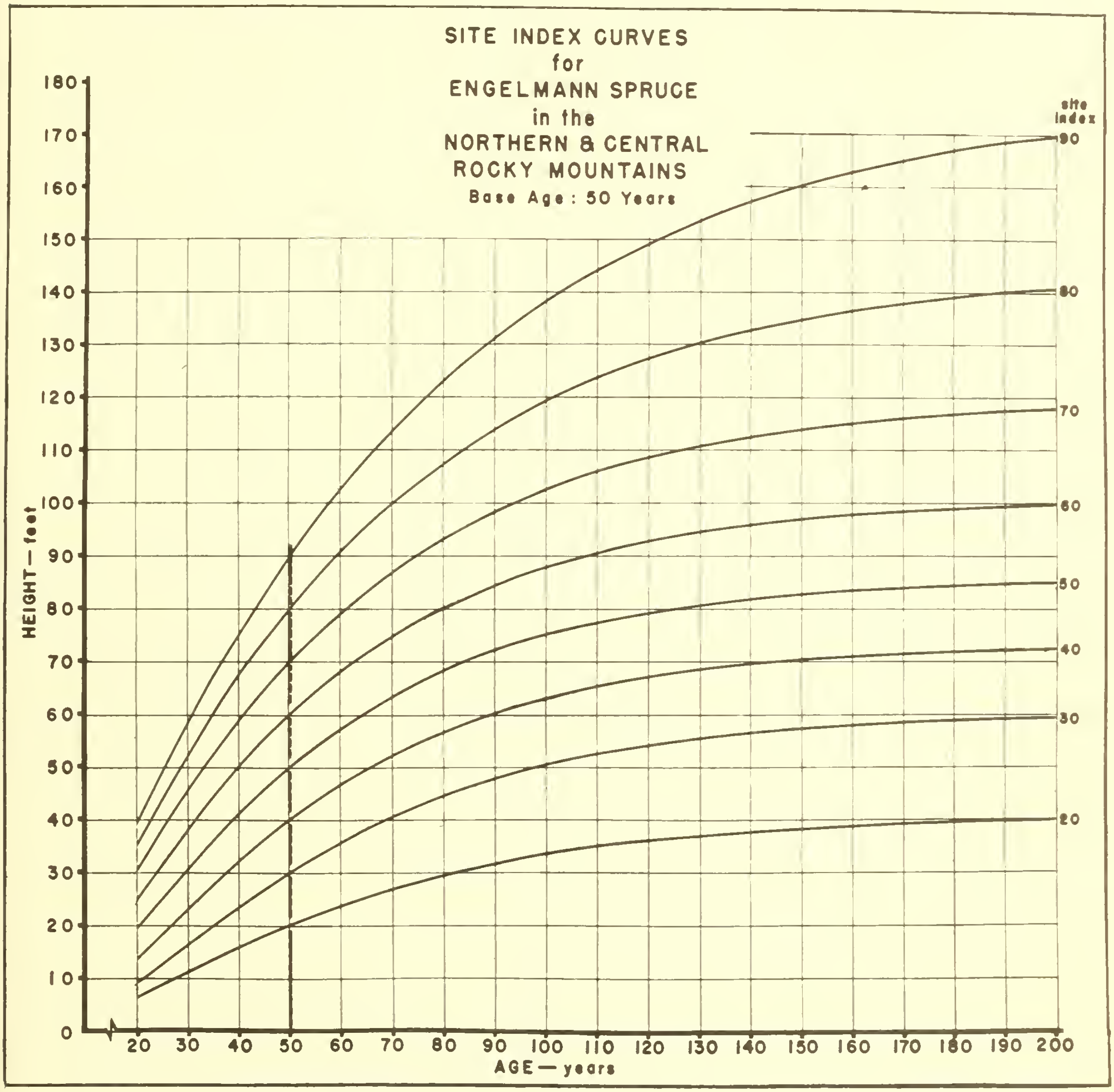

Figure 2.--Height on age curves at several levels of site index.

Table 1.--Height (in feet) of trees in the dominant stand by age and site index[1]

Total age (years)	Site index 20	30	40	50	60	70	80	90
20	6	9	14	19	25	30	35	40
30	11	16	23	31	38	46	52	58
40	16	23	32	41	50	59	67	75
50	20	30	40	50	60	70	80	90
60	24	36	47	57	68	79	91	103
70	27	41	52	63	75	87	100	114
80	30	45	57	68	80	93	108	123
90	32	48	60	72	85	99	114	132
100	34	51	63	75	88	103	119	138
110	35	53	65	77	91	106	124	144
120	36	54	67	79	93	109	128	149
130	37	56	69	81	95	111	131	154
140	38	57	70	82	96	113	133	157
150	39	57	70	83	97	114	135	161
160	39	58	71	84	98	115	137	163
170	39	59	72	84	99	116	138	165
180	40	59	72	85	99	117	139	167
190	40	59	72	85	100	118	140	169
200	40	60	73	85	100	118	141	170

[1] All height values are rounded to the nearest foot.

Table 2.--Standard error of mean plot site index to be expected from samples of given size

Sample size (no. of trees)	Standard error expected (feet): Idaho and Montana	Wyoming, Utah, Colorado, Arizona, and New Mexico
1	10.21	6.94
2	7.22	4.90
3	5.89	4.00
4	5.10	3.47
5	4.57	3.10
6	4.17	2.83
7	3.86	2.62
8	3.61	2.45
9	3.40	2.31
10	3.23	2.19
15	2.64	1.79
20	2.28	1.55
25	2.04	1.39

APPROXIMATIONS FOR ELECTRONIC COMPUTER PROGRAMS

The equation expressing tree height as a function of age and site index (as shown on page 7) cannot be solved explicitly for site index given age and height. Yet an equation expressing estimated site index as a function of age and height is desirable for incorporation in computer programs. Therefore, the following equation was derived to approximate the site index curves in an explicit form:

$$S = H+b_1X_1+b_2X_2+b_3X_3+b_4X_4+b_5X_5+b_6X_6+b_7X_7+b_8X_8+b_9X_9+b_{10}X_{10}+b_{11}X_{11} \quad (1)$$

where:

b_1	$= 0.10717283 \times 10^{2}$	X_1	$= (\ln A - \ln 50)$
b_2	$= 0.46314777 \times 10^{-2}$	X_2	$= [(10^{10}/A^5)-32]$
b_3	$= 0.74471147$	X_3	$= H[(10^4/A^2)-4]$
b_4	$= -0.26413763 \times 10^{5}$	X_4	$= H(A^{-2.5}-50^{-2.5})$
b_5	$= -0.42819823 \times 10^{-1}$	X_5	$= H(\ln A-\ln 50)^2$
b_6	$= -0.47812062 \times 10^{-2}$	X_6	$= H^2[(10^4/A^2)-4]$
b_7	$= 0.49254336 \times 10^{-5}$	X_7	$= H^2[(10^{10}/A^5)-32]$
b_8	$= 0.21975906 \times 10^{-6}$	X_8	$= H^3[(10^{10}/A^5)-32]$
b_9	$= 5.1675949$	X_9	$= H^3(A^{-2.75}-50^{-2.75})$
b_{10}	$= -0.14349139 \times 10^{-7}$	X_{10}	$= H^4[(100/A)-2]$
b_{11}	$= -9.481014$	X_{11}	$= H(A^{-4.5}-50^{-4.5})$

S = site index
H = total tree height
A = total tree age
ln is the natural logarithm, i.e., to base *e*.

When age is 50 years all terms in the equation come to zero, except the first, and at that age site index equals height. The standard error of estimate (Syx) for this equation is 0.689125 foot of site index units.

If a shorter, but less precise equation is desired, the following is recommended:

$$S = H+k_3X_3+k_4X_4+k_6X_6+k_9X_9+k_{10}X_{10} \quad (2)$$

where:

$k_3 = 0.32158242$

$k_4 = -0.98468901$

$k_6 = -0.12253415$

$k_9 = 1.0662061$

$k_{10} = -0.80894818$

and other symbols are as previously defined. For the abbreviated equation, Syx = 1.22469 feet.

These equations are valid for trees between the ages of 20 and 200 years and for site indices ranging from 10 to 95. Only 1 percent of trees have been found to reflect a site index higher than 75, so very few trees will be outside the range of the equations. If trees older than 200 years must be used as site trees, estimate the site index as if tree age were 200. Above that age, height growth has decreased to very little. Either of the two equations given can be used to estimate site index instead of using figure 2 or table 1.

OUTLINE OF MENSURATIONAL TECHNIQUE USED IN THIS STUDY

It is intended that the technique by which these curves were constructed will be described more thoroughly in a future publication. The information in this section indicates only the basis of these site index curves.

Polymorphism within the family of curves was achieved by a generalization of the method described by Osborne and Schumacher (1935). The generalization considers not only the trend of standard deviation, but also the skewness and kurtosis of residuals about the mean curve of height over age. This was done by dividing the site tree observations into 19 groups according to ascending values of total tree age. Each group contained about 100 paired measurements of age and height. Nine percentage points of the distribution of heights within each age group were estimated. These points represent the heights such that 1, 5, 10, 25, 50, 75, 90, 95, and 99 percent of the trees in each group were shorter than the respective limit. The estimates were based on tables of probability percentage points for skewed frequency distributions (Johnson, Nixon, and Amos 1963). When connected, the points established for a given level of probability, e.g., the 75-percent point, in the 19 groups of measurements form a curve of height over age. This can be assigned a site index by reading its height at the index age of 50 years. It can then be said that this, or greater, site index will be expected in one-fourth of all Engelmann spruce stands in the area from which the basic data were taken.

Nine curves of height over age, one for each probability level, were drawn by connecting the points for the respective probability level in each of the 19 age classes.

Several growth equations were fitted to these curves in an attempt to express the relationship between tree height and tree age. The most appropriate was found to be:

$$H^{1-m} = a^{1-m}\,[\,1-b\cdot\exp(-kA)\,] \tag{3}$$

where:

H = total tree height
A = total tree age
a, b, k, m are coefficients of the equation to be estimated by the method of least squares.
$\exp(-kA)$ means e, the base of Naperian logarithms, taken to the $(-kA)$ power.

This model and its use as a growth curve have been discussed in detail by Richards (1959).

Coefficients were estimated for the nine curves. Each of the nine equations which resulted was solved for height at age 50, thereby assigning a site index value to each set of 19 equiprobable points.

The height of trees in the dominant stand is not a function of tree age alone, but also of the site upon which the trees are growing. Site index was introduced into the tree height equation as an independent variable by expressing the coefficients of equation (3) as functions of site index in the following manner:

$$a = p_0+p_1S+p_2S^2+p_3S^3 \tag{4}$$

$$k = \exp(q_0+q_1S+q_2S^2) \tag{5}$$

$$m = \exp(r_0+r_1\ln S+r_2\ln^2 S) \tag{6}$$

where:

S = site index
a, k, m are coefficients of equation (3), and p_i, q_i, r_i are coefficients to be estimated in the multivariate equation.

Because the height of all trees must be zero at age zero, this constraint was placed on the curves by holding the coefficient $\underline{b}$ in equation (3) constant at the value of 1.0 or, in other words, eliminating it from

the equation. The introduction of site index into the multivariate height equation as shown in equations (4), (5), and (6) allows the resulting site curves to be polymorphic.

The multivariate function was then fitted to the nine sets of equiprobable points, with site index for each set entered as an independent variable. The final equation used to express tree height as a function of age and site index is:

$$H^{1-m} = a^{1-m}[1-\exp(-kA)]$$

with the coefficients,

$$a = 26.38029+4.545548S-0.070759S^2+0.000501S^3 \quad (7)$$

$$k = \exp(-3.997462+0.01532S-0.000183S^2) \quad (8)$$

$$m = \exp(-9.603352+5.405062\ln S-0.817504\ln^2 S) \quad (9)$$

The curves of figure 2 are described by this equation. Table 1, showing tree height according to age and site index, was obtained by repeated solution of the equation.

TESTS OF THE FINAL SITE INDEX CURVES

An essential test of any system of site index curves involves the independence of stand age and estimated site index. If the two are not independent, i.e., if correlation exists, then estimated site index can be expected to change as stand age increases. This has been an undesirable feature of some anamorphic site index curves constructed according to the so-called strip method (Chapman and Meyer 1949; Watt 1960).

Anamorphic site index curves may, by their proportional nature, bring about a correlation between stand age and estimated site index. On the other hand they may not if the true curves of height over age, which we seek to approximate by various mensurational techniques, are really proportional from one level of site quality to another. Such a situation seldom occurs, however. It is advisable to use a mathematical model that can express polymorphic curves, yet can express proportional curves if doing so is necessary to obtain the least-squares fit to the data.

Correlation between age and estimated site index may also be caused by patterns of land use in the past. Curtis (1964) found this to occur with some tree species in New England. In this study it has been assumed that previous land use patterns have not operated to produce such an effect for the following reasons:

1. The data were drawn from a wide geographic area embracing several different economic regions.

2. Most of the data came from National Forests where cutting practices have been rather different from those on smaller private holdings.

3. The effects of cutting practices and land use, if any, have been felt for at least 2 centuries' less time in the Mountain States than in New England.

4. The Engelmann spruce forest type is, in general, located at higher elevations which are some distance removed from agricultural areas.

Beyond the logic of assuming the independence of estimated site index from age, it is possible to test this assumption by the statistical test which will be described.

To test the curves presented in this paper, a site index value was assigned to each of the 1,928 site trees in the basic data. This was also done for a sample of site trees from Arizona and New Mexico, which was obtained too late to be included in the data used in construction of the curves. The site trees were then sorted into six groups, based on the geographical area from which they were drawn. The number of trees in each of the subregional groups was:

Subregion	I	Northern Idaho and western Montana	194
Subregion	II	Central Idaho	138
Subregion	III	Southern Idaho, northern Utah, and western Wyoming	437
Subregion	IV	Colorado and eastern Wyoming	807
Subregion	V	Southern Utah	352
Subregion	VI	Arizona and New Mexico	133

Within each areal group and within all groups combined, an attempt was made to find significant trends of site index with tree age by fitting the equation:

$$S = b_0+b_1A+b_2A^2+b_3A^3 \tag{10}$$

to the joint distribution of tree age (A) and estimated site index (S). All multiple correlation coefficients thus obtained failed to indicate significant difference from zero at the 95-percent probability level. Therefore, the hypothesis that there is correlation between stand age and estimated site index was rejected and the two were assumed independent. It should be borne in mind that one cannot claim absolute certainty for any statistical test; at best, only a high probability of being correct can be claimed. In this case, the probability of this assumption's being correct seems quite high.

For the data from Arizona and New Mexico, as for those from other areas, no statistically significant trend of estimated site index with age was observed. This indicates that on the basis of the available data these curves are acceptable for use in Arizona and New Mexico as well as in the other Mountain States. However, because no data used in constructing this system of curves came from Arizona and New Mexico, their title expresses validity only for the northern and central Rocky Mountains.

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